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THE INFLUENCE OF A VARIABLE DEPTH ON STEADY ZONAL BAROTROPIC FLOW

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Gene H. Porter, LT. U.S.N.

M.S. Thesis

Thesis
P742

THE INFLUENCE OF A VARIABLE DEPTH ON STEADY ZONAL BAROTROPIC FLOW

by

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A thesis submitted in partial fulfillment

of the requirements for the degree of

MASTER OF SCIENCE

UNIVERSITY OF WASHINGTON

1963

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ACKNOWLEDGMENTS

The author wishes to express his gratitude to the faculty and staff of the Department of Oceanography, University of Washington, without whose assistance this work would not have been possible. Special thanks are due Dr. Maurice Rattray, Jr. for his continuing encouragement and advice during the course of this investigation.

This research was performed while the author was on active duty with the United States Navy as a participant in the Junior Line Officers Advanced Scientific Education Program (Burke Program) administered by the Bureau of Naval Personnel.

ABSTRACT

The barotropic vorticity equation is utilized to investigate the steady topographic disturbance of an initially uniform zonal current. Solutions are obtained on the beta-plane for various discontinuous bathymetric profiles. It is shown that a smooth slope does not appreciably modify the results. Stability criteria are derived that define the values of the flow parameters for which the streamlines become discontinuous.

The solutions are evaluated for several observed disturbances of oceanic currents. The velocities implied by the observed wavelengths are physically realistic, but the discrepancy between observed and calculated amplitudes suggests that the vertical structure of the velocity profile and the baroclinicity of the ocean are important.

CHAPTER I

INTRODUCTION

In most studies of the large scale circulation patterns of the oceans, the sea floor is assumed to be either flat or infinitely deep. Such simplifications are made to facilitate the investigation of certain classes of flows in which it is suspected that topographic effects are nonexistent or of minor importance. That currents in the real ocean are to some extent topographically affected has been pointed out by many authors. Perhaps the most universally cited example of such an effect is the wave-like structure in the dynamic topography of the Antarctic Circumpolar Current (ACC) where it crosses each of several ridges (see Figure 1).

Many contributions to the understanding of oceanic currents have been achieved through the application of the techniques and principles of dynamic meteorology. That many apparent similarities exist between oceanic and atmospheric flows was graphically demonstrated in 1951 when Fuglister and Worthington published the results of an intensive multiple ship survey of a Gulf Stream meander. Their illustration of the motion and incipient occlusion of the meander bears a striking resemblance to an earlier illustration (Berggren, et al., 1949) of the unstable waves observed at the 500 mb level in the atmosphere. It will therefore be found instructive to consider briefly the results of previous investigations of topographic effects on both oceanic and atmospheric flows.

I-1. Oceanographic Investigations

Ekman (1923), in an attempt to account for the deflection of the Gulf Stream over the Southeast Newfoundland Ridge, considered the effect of bathymetric relief on a homogeneous current under a balance of pressure gradient, bottom friction, and constant Coriolis forces. His results show the current deflection to be dependent on the slope of the bottom, but not the total depth, and give a net lateral displacement of the streamlines (to the right for flow over a ridge in the Northern Hemisphere). For flow over successive ridges, Ekman's streamlines are 90 degrees out of phase with the topography, the maximum deflection occurring at the point of inflection of the bottom profile.

In 1932 Ekman reconsidered the problem and included accelerations in addition to bottom friction, constant Coriolis force, and pressure gradients. Subject to the requirements that both the bottom slope and the variation in depth be small, he obtained a solution that gives streamlines in phase with the bathymetry, the maximum displacement to the right occurring over the point of minimum depth.

In an attempt to rigorously investigate the effect of bottom friction on a topographically disturbed homogeneous current, Görtler (1941) utilized Prandtl's boundary layer theory. He simplified the problem by considering a constant Coriolis parameter and a variation in depth that is small compared to the total depth of the fluid. His solution to the linearized equations reduces to that of Ekman (1932) provided that Görtler's friction parameters, including the boundary layer thickness, are properly identified with Ekman's "depth of frictional resistance." Görtler extended Ekman's solution by including the possibility of the upstream flow approaching the topographic feature at other than normal incidence. He also

evaluated the frictional effects for several physically reasonable values of the flow parameters and thereby demonstrated that such effects are negligible for most atmospheric and oceanic currents. Görtler pointed out though, as did Ekman (1932), that topographic effects in which inertia plays an important role may not properly be treated as independent of planetary effects when the scale of motion is such that the latter must be considered. This point is central to the investigation at hand.

Sverdrup (1941) took a new approach to the problem of topographic effects. He neglected friction and considered that the effect of topographic features was to disturb the internal distribution of mass. Being unable to treat the problem rigorously, he nevertheless was able to demonstrate qualitatively that a bathymetric feature which rises from a layer of no motion into an internal solenoidal field and its associated current will disturb that current in such a way that the physical sea surface will be depressed above the feature. This effect causes the current to deviate to the right (in the Northern Hemisphere), the maximum deviation being over the point of minimum depth, and then to return to its original configuration downstream. That the converse is true for flow over a depression is not clear, as there is no mechanism for disturbing the solenoidal field. Sverdrup's conclusions agree qualitatively with the observed features of the circulation in the Southern Ocean.

Shtokman (1947), in a discussion of his subsequently published (1948b) theory of topographic effects on ocean currents, demonstrated an apparent serious discrepancy between the theories of Ekman, Görtler, and Sverdrup and observed flow features of the South Atlantic and Caspian Sea. Considering Defant's (1941) chart of the "absolute" topography of the surface in the Atlantic, and relating this chart to the bathymetry of the South Atlantic

between 25° and 50° South, Shtokman observed that the eastward currents are deflected poleward over shoaling water and equatorward over deepening water. This is contrary to the observed deflection of the ACC. Since the direction of deflection in the earlier theories depended directly on the sign of the Coriolis parameter, there was no mechanism available to explain opposite deflections in the same hemisphere.

Noting the above apparent discrepancy between theory and observation, Shtokman inferred that the Coriolis parameter, if considered constant, should not be the primary consideration. On this basis, he adapted an earlier (1948a) derivation of the equations relating wind stress to lateral friction to the problem of a variable depth. For zero wind stress, the signs of the terms in his equations give qualitative agreement with Sverdrup's (1941) conclusions. Shtokman's approximation of the complete equations including wind stress implies that the direction of the current deflection over a topographic feature depends on the sign of the local curl of the wind stress and not on the hemisphere. Thus, for shoaling water, a negative wind stress curl yields a deflection to the right, and vice versa.

As Shtokman pointed out, his results agree with the observations he cited. Thus the absence of perturbations on Defant's 1941 topography of the surface of the South Atlantic between 40° and 50° South is explained by the existence of maximum wind stress, and hence zero wind stress curl, in these latitudes. A later presentation by Defant (1961) of the dynamic topography of the same area indicates substantially the same structure except for the addition of an anticyclonic eddy at approximately 45°S - 20°E (see Figure 2). If this is indeed an area of zero wind stress curl, then Shtokman's theory does not account for this eddy.

The writer believes that a more fundamental objection to Shtokman's theory may be made on the basis of his method of introduction of a variable depth. The depth which Shtokman ultimately considers as representing the actual bathymetry was originally defined as that depth at which the transport components vanish. Charney (1955) and Fofonoff (1962) have shown that this depth of no motion is a function of the transport itself, and hence is of a nature that is fundamentally different from the bathymetry.

Shtokman's refutation of the theories of Ekman (1923), Görtler (1941), and Sverdrup (1941) based upon consideration of the near shore currents in the Caspian Sea seems inappropriate in that those earlier theories specifically excluded changes of depth that amount to a significant fraction of the total depth. The earlier theories also excluded flows initially parallel to the isobaths, such as are considered by Shtokman (1947, Fig. 5).

Neumann (1960) attempted to combine analytically the results of Ekman (1923) and Sverdrup (1941). Neumann initially considered the steady flow in a baroclinic ocean to be maintained by a balance of Coriolis, friction (both wind stress and "internal" friction), and pressure gradient forces. He pointed out, as have Defant (1941) and others, that for homogeneous water, the variation of Coriolis parameter with latitude requires that all purely geostrophic currents be zonal, unless the depth increases from the equator toward the poles in proportion to the sine of the latitude. Neumann obtained approximate solutions for several classes of flow, including that of homogeneous water flowing over a ridge under the influence of a variable Coriolis parameter. This latter solution gives a Sverdrup-type deflection of the streamlines without the consideration of a solenoidal field. Neumann's solutions are based upon the requirement that the change

of depth be small compared to the total depth, but the numerical examples given do not clearly meet that criterion.

Saint-Guily (1962) has extended his earlier (1959) treatment of the generalized Ekman problem to the case of a variable depth. Subject to the assumptions of vanishing friction and moderate changes in depth, Saint-Guily obtained solutions for the steady zonal flow of homogeneous water over a sill that correspond to those of Neumann (1960) for a variable Coriolis parameter. Saint-Guily's initial disregard of the field acceleration terms in the equations of motion restricts the validity of his solutions to regions where the ratio of the relative vorticity to the Coriolis parameter (Rossby Number) is small.

I-2. Meteorological Investigations

As was noted above, topographic effects on ocean currents have parallels in meteorological problems. Queney (1949) summarized previous theoretical treatments of topographic effects, including the results of his own extensive treatment of the problem (1947) utilizing perturbation techniques. Queney considered the stability associated with the density stratification of an infinitely deep atmosphere, and it is therefore difficult to apply his results directly to a homogeneous ocean. Charney and Eliassen (1949) concluded that the real gravitationally stable atmosphere may be treated as an "equivalent barotropic atmosphere" whose properties coincide with those of the real atmosphere at a height of zero horizontal divergence (approximately 500-700 mb). Charney and Eliassen were able to substantially improve their numerical prediction techniques by including a rough model of the effects of continental topography. The importance of such continental effects in the study of large scale atmospheric flows has been discussed by Bolin (1950). He noted the regular perturbations of the

pressure surfaces that occur on monthly (and longer) mean weather charts of both the Northern and Southern Hemispheres and argued in favor of a topographic mechanism for the generation of such perturbations as opposed to a thermodynamic mechanism. Bolin linearized the equations and obtained approximate solutions for flow over an isolated mountain.

Oi (1956) considered the problem of the frictionless barotropic flow of air over a mountain range under the influence of a constant Coriolis parameter. He was able to include accelerations in a manner similar to Ekman (1932) and, again for small changes in the total depth of the fluid column, obtained approximate solutions showing the effect of a mountain range on an initially zonal flow.

I-3. Initial Evaluation of the Problem

All of the above investigations, with the exception of that of Shtokman, have essentially linearized the problem by considering only small changes in relief relative to the total depth of the fluid column. This may be shown (Bolin, 1950) to be equivalent to assuming that the relative vorticity of the fluid is small compared to the local value of the Coriolis parameter. In the ocean, with a mean depth of about 4 km, and in the equivalent homogeneous atmosphere, with a height of 8 km, it is apparent that there are many areas where such a linearization will be inappropriate. At low latitudes the relative vorticity may certainly approach or exceed the value of the Coriolis parameter. In the oceans there are innumerable submarine features, covering a spectrum of size scales, that rise well above the mean surrounding depths. These observations on the inherent limitations of the perturbations techniques, coupled with the well known importance of the variation of the Coriolis parameter with latitude, provide the basis for the present investigation.

In subsequent chapters, Görtler's (1941) conclusions will be accepted and it will be assumed that the flow of the barotropic ocean is frictionless, although it is recognized that friction must be important in some limiting sense. Indeed, Görtler pointed out that frictional effects may become important when the horizontal scale of the topographic feature under consideration is very much larger than the inertial radius of the flow. As the objective of this investigation is to study the combined effects of the variation of Coriolis parameter and finite changes in depth over a range of scales, subsequent assumptions will not restrict these essential features of the problem.

The concept of vorticity has become fundamental to the investigation of both atmospheric and oceanic flows. In its general form, the vorticity equation is a concise statement of one of the laws of fluid motion. For the class of flow under consideration its form facilitates investigation from either an Eulerian or Lagrangian viewpoint. This property of the vorticity equation will be utilized in succeeding chapters.

The deep currents in the oceans will be treated as though they were barotropic, not because this mode is necessarily strictly representative of in situ deep currents, but because the barotropic equations of motion are simpler to treat exactly than are the baroclinic equations. The influence on this barotropic flow of topographic features with the scales of features existing in the real ocean will be investigated.

Topographic effects observable in the baroclinic flow (see Figure 1) must be related to those occurring in the barotropic flow, but this relation has not yet been determined.

CHAPTER II

FORMULATION OF THE PROBLEM

II-1. Basic Equations

We consider the horizontal components of the equation of motion of a frictionless, homogeneous ocean or atmosphere,

$$\frac{Du}{Dt} - fv = -\frac{1}{\rho}\frac{\partial P}{\partial x}, \qquad \frac{Dv}{Dt} + fu = -\frac{1}{\rho}\frac{\partial P}{\partial y}, \tag{1}$$

together with the continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{\partial w}{\partial z}, \tag{2}$$

where u, v, and w are the velocity components along the x (eastward), y (poleward), and z (upward) axes. The Coriolis parameter (equal to twice the local vertical component of the earth's rotation) is denoted by f, and P is the local pressure. The local acceleration of gravity is g and ρ is the fluid density, both of which are assumed constant.

II-2. The Vorticity Equation

The two components of equation (1) are cross-differentiated with respect to y and x respectively, and then subtracted to give, after rearrangement of terms,

$$\frac{D(\zeta + f)}{Dt} = -(\zeta + f)\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right] + \left[\frac{\partial w}{\partial y}\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\frac{\partial v}{\partial z}\right], \tag{3}$$

where $\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$, the vertical component of relative vorticity.

Equation (3) may be considered as the primitive vorticity equation, where the change in the absolute vertical vorticity of a parcel of fluid is

seen to be due to the local two-dimensional velocity divergence and the conversion of horizontal vorticity to vertical vorticity.

If we now consider u and v to be independent of z, the vortex tube term in equation (3) drops out, and we may integrate equation (2) from the bottom of a column to the free surface to obtain

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = - \frac{1}{(h + \eta)} \frac{D(h + \eta)}{Dt} ,$$

where η is the elevation of the free surface above its undisturbed position, and h is the height of the undisturbed free surface above the rigid bottom, or ground. Substitution of the above equation into equation (3) then yields

$$\frac{D}{Dt} \left[\frac{f + f}{h + \eta} \right] = 0 , \qquad (4)$$

the equation commonly interpreted as expressing the conservation of potential vorticity along a trajectory. It is of interest to note that no assumption of steady state has been made in the preceding development.

II-3. The Transport Streamfunction

To facilitate the introduction of the volume transport streamfunction, the vertically integrated continuity equation may be written as

$$\frac{\partial}{\partial x}\left[u(\eta + h)\right] + \frac{\partial}{\partial y}\left[v(\eta + h)\right] = - \frac{\partial(\eta + h)}{\partial t} = - \frac{\partial \eta}{\partial t} .$$

For stationary conditions this equation is satisfied identically by a volume transport streamfunction defined such that

$$\frac{\partial \Psi}{\partial y} = - u(\eta + h), \text{ and } \frac{\partial \Psi}{\partial x} = v(\eta + h); \text{ where } \Psi = \Psi(x,y). \qquad (5)$$

Henceforth, we will neglect the elevation of the free surface η compared to the depth h, both as a readily justifiable approximation and for expediency.

In terms of the streamfunction ψ, the first integral of the vorticity equation, equation (4), is then

$$\frac{\nabla\cdot(\frac{1}{h}\nabla\psi) + f}{h} = \text{constant (along a streamline)}. \qquad (6)$$

Charney (1955) and Fofonoff (1962) have shown that there must be a functional dependence of the Coriolis parameter and the depth h on the streamfunction ψ. This may be readily seen by considering that the above equation is applicable along an arbitrary streamline, and that the choice of a specific streamline then determines the geographic path followed by a fluid column and hence its future depth and latitude. Substantial analytical difficulties arise when an attempt is made to express both the bathymetric relief and the variable Coriolis parameter as functions of the upstream flow pattern.

CHAPTER III

SOLUTION FOR ZONAL FLOWS

The difficulties mentioned at the end of Chapter II may be alleviated by restricting our consideration to flows that are strictly zonal upstream of the region of interest, and to topography that is independent of y.

III-1. Simplification of the Problem

Specifically, we now establish the following conditions: at some point upstream of the region of interest, say at $x = x_o$, the flow is defined by $\Psi = -UHy$, where H is a constant depth and U is the transport velocity (positive eastward); in the first region of interest, say $x \geq x_o$, the depth is a function of x only, h(x). In all regions the Coriolis parameter is proportional to y, the coefficient of proportionality being β, and with the origin of the y axis at the equator, $f = \beta y$. Denoting the upstream region by the subscript 1, we then have $\Psi_1 = -UHy_1$, hence $f_1(\Psi_1) = \beta y_1(\Psi_1) = -\beta\Psi_1/UH$. Substituting into equation (6),

$$-\frac{\beta\Psi_1}{UH^2} = \frac{\nabla\cdot(\frac{1}{h}\nabla\Psi) + \beta y}{h(x)} , \qquad (7)$$

where $\Psi = \Psi_1$ (because equation (6) applies along a streamline). Rearranging and dropping the now superfluous subscript 1,

$$h(x)\nabla^2\Psi - \frac{dh}{dx}\frac{\partial\Psi}{\partial x} + \frac{\beta h^3(x)}{UH^2}\Psi = -h^2(x)\beta y . \qquad (8)$$

III-2. Matching Conditions

The flow defined by equation (8) must satisfy continuity requirements, that is, it must match the upstream transport at $x = x_o$. As the upstream transport is geostrophic, we might meet these requirements by matching meridional pressure gradients, but this is found equivalent to directly matching total transport, or transport components. The matching conditions are then

$$\left.\begin{aligned} \frac{\partial \Psi}{\partial x} &= \frac{\partial \Psi_1}{\partial x} = 0 \\ \frac{\partial \Psi}{\partial y} &= \frac{\partial \Psi_1}{\partial y} = -UH \end{aligned}\right\} \text{ at } x = x_o \,. \tag{9}$$

III-3. The General Solution

The vorticity equation (8) may readily be reduced to an ordinary differential equation by utilizing the separation $\Psi(x,y) = y\,\phi(x)$. Upon substitution and cancellation of y, equation (8) becomes

$$h(x)\phi''(x) - h'(x)\phi'(x) + \frac{p^2h^3}{H^2}\,\phi = -h^2\beta \tag{10}$$

where $p^2 = \beta/U$.

The matching conditions (9) become

$$\begin{aligned} \phi(x_o) &= -UH \\ \phi'(x_o) &= 0 \end{aligned} \tag{11}$$

To facilitate the solution of (10) subject to (11) we make the following transformation of the independent variable x:

Let $\phi(x) = G(s)$, where $s = \frac{p}{H}\int_{x_o}^{x} h(x)dx$, and $s_o = s(x_o) = 0$,

and define D(s) by: $h(x) = -UH^2D(s)$.

Upon substitution, equation (10) becomes

$$G_2'' + G_2 = 1/D_2(s) \tag{12}$$

and the matching conditions (12) becomes

$$\begin{aligned} G_2(0) &= G_1(0) = - UH \\ G_2'(0) &= G_1'(0) = 0 \ , \end{aligned} \tag{13}$$

where the subscript 2 indicates the second region considered, that is, the region immediately downstream of the uniform zonal flow. Anticipating that we may desire to consider more than one downstream region, we may immediately generalize equations (12) and (13) to:

$$G_n'' + G_n = 1/D_n(s) \tag{14}$$

and

$$\begin{aligned} G_n(s_{no}) &= G_{n-1}\left[s_{n-1}(a)\right] \\ G_n'(s_{no}) &= G_{n-1}'\left[s_{n-1}(a)\right], \end{aligned} \quad (n = 2, 3, 4, \ldots.) \tag{15}$$

where $$s(a) = \frac{p}{H}\int_{x_o}^{a} h(x)dx \ .$$

The complete solution of equation (14) is

$$G_n(s_n) = A_n \cos s_n + B_n \sin s_n + \int_{s_{no}}^{s_n} \frac{\sin(s_n - t)}{D_n(t)}\, dt \tag{16}$$

where the coefficients A_n and B_n are to be determined from equations (13) and (15).

The solution (16) may be evaluated (if only numerically) for any bathymetric profile that is independent of y. It represents the topographic disturbance of a zonal current subject to the following assumptions:

1. The ocean (or atmosphere) may be idealized as homogeneous with a free upper surface.
2. Horizontal velocities are invariant with depth.
3. The influence of friction is negligible.
4. The convergence of the meridians may be neglected, and the meridional variation of the Coriolis parameter may be approximated as a linear function of distance from the equator (β-plane approximation).
5. Displacements of the free surface from a level surface are small compared to the total height of the fluid column.
6. The initial flow is zonal with zero relative vorticity.
7. The flow, and hence the pressure field, is stationary.

CHAPTER IV

EVALUATION OF THE SOLUTION FOR SOME SIMPLE PROFILES

When solution (16) is transformed back into $\phi(x)$, it is apparent that eastward flow over a varying depth is periodic, while westward flow is exponential, as p is real or complex depending on the sign of U. Throughout this chapter, only eastward flow will be considered.

IV-1. The Bottom Profile is a Simple Step

We consider first a simple step-shaped feature in the topography. For $x < 0$, $h_1 = H$; for $x \geq 0$, $h_2 =$ constant. Then, transforming (13) and (16) back in terms of x, we have $s = r_2 px$, where $r_2 = h_2/H$. and $A_2 = -UH$, $B_2 = 0$. These values of the parameters give

$$\Psi(x,y) = y\,\phi(x) = -\frac{UHy}{r_2}\,(1 - (1 - r_2)\cos r_2 px)\ . \qquad (17)$$

It is apparent that $\phi(x)$ will vanish at some point for $r_2 \geq 2$. We may describe this situation as unstable because at such a value of x there will be no zonal flow. The characteristics of solution (17) are best illustrated by reducing all streamlines to the single path given by

$$-\frac{UHy}{\Psi} = \frac{r_2}{(1 - (1 - r_2)\cos r_2 px)}\ .$$

The left hand member of equation (18) is shown plotted against $r_2 px$ (Figure 3), for the stable case $r_2 = 0.5$ (Figure 3a), and for the unstable case $r_2 = 2.5$ (Figure 3b). In the unstable case the dashed streamline has been included to indicate the cellular structure extending to infinity at periodic values of x. This point will be further discussed in a subsequent chapter.

It is instructive to make an estimate of the wavelength of the specific stable flow shown in Figure 3a. Taking $\beta = 2 \times 10^{-13} cm^{-1} sec^{-1}$, and arbitrarily choosing 1 cm/sec as a characteristic velocity of the deep oceanic circulation (only the order of magnitude of the velocity is important, the wave-length depending upon its square root), the wave-length is on the order of 280 km. For an atmospheric velocity of 20 m/sec, the wavelength would be on the order of 12,000 km, that is, of continental dimensions.

IV-2. The Bottom Profile Resembles a Plateau

We next consider a more complex example, that of flow over a plateau (or over a flat bottomed, vertical sided valley). We establish the conditions that for $x < 0$, $h = H$ as before, and for $0 < x < a$, $h = h_2$. For $x > a$, $h_3 = H$, although the treatment is as easy with the final depth different from the initial depth.

From the previous example,

$$A_1 = B_1 = B_2 = 0; \quad A_2 = - UH,$$

and from the above definition of the topographic feature,

$$r_1 = r_3 = 1; \quad r_2 = h_2/H .$$

The boundary conditions on $\phi_3(x)$ are then

$$\begin{aligned} \phi_3(a) = \phi_2(a) &= - \frac{UH}{r_2} (1 - (1 - r_2)\cos r_2 pa) , \\ \phi_3'(a) = \phi_2'(a) &= - UHp (1 - r_2) \sin r_2 pa . \end{aligned} \tag{19}$$

From equation (16), transforming back in terms of x,

$$\phi_3(x) = (A_3 + UH)\cos p(x - a) - UH + B_3 \sin p(x - a) . \tag{20}$$

Solving equations (19) and (20) for A_3 and B_3 and substituting back into equation (20) we obtain

$$\Psi_3(x,y) = y\,\phi_3(x) = -UHy\left[(1/r_2 - 1)(1 - \cos r_2 pa)\cos p(x - a) + (1 - r_2)\sin r_2 pa \sin p(x - a) + 1\right] \tag{21}$$

Equation (21) describes the flow for all $x > a$ and for r_2 either greater than or less than unity.

In our analysis of equation (21) we will only consider values of the parameters r_2 and a that give stable flow in region 2. It does not seem realistic to match a solution that is unstable with a subsequent downstream flow.

Proceeding now to the development of stability criteria for the downstream flow, by our earlier definition instability occurs for some value of x when the bracketed term in equation (21) vanishes, or:
$(1/r_2 - 1)(1 - \cos r_2 pa)\cos p(x - a) + (1 - r_2)\sin r_2 pa \sin p(x - a) = -1.$
If we now let the coefficients of $\cos p(x - a)$ and $\sin p(x - a)$ be represented by $C_1(r_2,a)$ and $C_2(r_2,a)$ respectively, and divide the above equation by $(C_1^2 + C_2^2)^{\frac{1}{2}}$ we obtain

$$\frac{C_1}{(C_1^2 + C_2^2)^{\frac{1}{2}}}\cos p(x - a) + \frac{C_2}{(C_1^2 + C_2^2)^{\frac{1}{2}}}\sin p(x - a) = \frac{-1}{(C_1^2 + C_2^2)^{\frac{1}{2}}} .$$

By defining the angle ϕ such that $\tan \phi = C_1/C_2$, and utilizing a simple trigonometric identity, this equation can be written

$$(C_1^2 + C_2^2)^{\frac{1}{2}} \sin(\phi + px - pa) = -1 .$$

Noting that ϕ and pa are phase angles, this equation will be satisfied at some value of x only if $C_1^2 + C_2^2 \geq 1$.

In terms of r_2 and a, the criterion for the instability of the downstream flow is

$$(1 - \cos r_2 pa)^2 + r_2^2 \sin^2 r_2 pa \geq \left[\frac{r_2}{1 - r_2}\right]^2 . \qquad (22)$$

No further general simplification of the criterion is possible, equation (22) being transcendental in r_2. However it would appear that there is no value of a for which an r_2 cannot be found that will satisfy the above relation. As must be true, there is no instability for $r_2 = 1$. A rough calculation indicates stability for r_2 between 0.67 and 2. As both r_2 and a are topographically determined, the properties of the solution are best illustrated by example.

We first investigate the flow when $r_2 = 0.5$, previously found to be stable throughout region 2. The solution (21) for the downstream region may then be written

$$-\frac{UHy}{\Psi} = \left[1 + (1 - \cos 0.5\ pa) \cos p(x - a) + 0.5 \sin 0.5\ pa \sin p(x - a)\right]^{-1}. \qquad (23)$$

Four representative breadths a will be chosen. The first, $0.5\ pa = \pi/3$ (or $7\pi/3$, etc.) locates the down-step in a region of S.E. flow; the second, at $0.5\ pa = 5\pi/3$ locates the step in a region of N.E. flow. The third breadth is chosen such that the step is at the point of maximum excursion, $0.5\ pa = \pi$; and the fourth at the point where the flow has regained its original latitude, $0.5\ pa = 2\pi$.

Figure 4 shows the normalized streamline for the first case. The flow over the plateau is the same as that shown in Figure 3a, but the zonal axis has been expanded to be compatible with subsequent examples. The characteristic "wavelength" associated with this and subsequent examples is half that estimated earlier for flow on a plateau, that is, all stable

downstream wave-like motions have a characteristic wavelength equal to that of a stationary Rossby Wave.

Figure 5 shows the normalized streamline for the second case. The next example, as shown in Figure 6, indicates the same type of instability previously found for a single step with $r_2 > 2$. For the assumed $r_2 = 0.5$, this type of instability exists for all 0.5 pa between $\pi/2$ and $3\pi/2$.

Figure 7 shows the undisturbed downstream flow pattern that results when the plateau is an integral number of wavelengths long. This is the only topographic condition that allows the downstream flow to proceed undisturbed.

IV-3. Other Discontinuous Bottom Profiles

We next consider the case where the flow is over a valley such that $r_2 = 1.5$. The stability criterion (22) shows that such a flow is stable for all values of a. Figure 8 indicates the flow regime with $pa = 5\pi/3$. The flow for other choices of pa is similar but with different amplitudes. Positioning the step at integral multiples of $2\pi/3$ gives an undisturbed downstream flow. The wavelengths of the disturbances over the valley are of course two-thirds the wavelength of the downstream waves.

Returning now to the general solution, we recall that the final depth was chosen equal to the initial depth solely for convenience. We may rewrite the solution (21) to include a final depth h_3 = constant, where $r_3 = h_3/H$;

$$\Psi_3(x,y) = -\frac{UHy}{r_3}\left[1 + (1 - r_2)\sin r_2 pa \sin r_3 p(x - a) - \left\{(1 - r_2)\cos r_2 pa + (r_2/r_3 - 1)\right\}\frac{r_3}{r_2}\cos r_3 p(x - a)\right] \tag{24}$$

For $r_3 < r_2 < 1$, the flow is over two successive shoaling steps, and conversely, for $r_3 > r_2 > 1$, the flow is over two successively deepening steps. Intermediate alternatives represent modifications of the earlier examples.

IV-4. The Bottom Profile is Smoothly Sloping

Over any region of constant depth, the wavelength of the disturbance is fixed by the upstream velocity, the relative depth, and β. In order to investigate the possible influence of the mode of transition from one region of constant depth to another on the amplitude of the disturbance, it will be instructive to consider flow over a gradually shoaling slope. Returning to the general equations (10) through (16), we define the depth in the transition region as: $h(x) = Hx_o/x$, for $x \geq x_o$. We are free to choose any numerical value of x_o that gives a desired slope. For simplicity we choose $px_o = 1$, which gives an x_o on the order of 100 km. This in turn describes a slope that halves the depth in about 100 km. Substitution then gives the transformed variables

$$D(s) = \frac{-e^{-s}}{UH} , \quad s = \ln x/x_o , \quad s_o = 0 .$$

The solution of equations (12) and (15) is then

$$G_2 = -\tfrac{1}{2}UH(e^s + \cos s - \sin s) . \tag{25}$$

In terms of the original variables, the flow over the slope has the form

$$\psi_2(x,y) = -\tfrac{1}{2}UHy \left[x/x_o + \cos(\ln x/x_o) - \sin(\ln x/x_o)\right] . \tag{26}$$

If $px_o \neq 1$, the solution is

$$\psi_2(x,y) = \frac{-UHy\, px_o}{1 + p^2x_o^2} \left[px + \frac{\cos(px_o \ln x/x_o)}{px_o} - \sin(px_o \ln x/x_o)\right] . \tag{27}$$

We now match solution (26) to flow on a flat plateau. We choose a relative depth of 0.4, which requires that we terminate solution (26) at $x = 2.5\ x_o$. The matching conditions are then

$$\phi_2(2.5x_o) = \phi_3(2.5x_o) = A_3 = -1.15\ UH$$

$$\phi_2'(2.5x_o) = \phi_3'(2.5x_o) = r_3 p B_3 = -0.22\ pUH;\ B_3 = -0.55\ UH.$$

The complete solution is

$$\Psi_3(x,y) = y\,\phi_3(x) = -UHy\left[2.5 - 1.35\cos r_3 p(x-2.5x_o) + 0.55\sin r_3 p(x-2.5x_o)\right]. \tag{31}$$

Figure 9 shows equation (31), again as a normalized streamline. Plotted on the same figure is equation (17), with $r = 0.4$. It is apparent that a transition slope of the above dimensions causes a phase shift but does not significantly change the amplitude of the deflection, which in this example is only 3 percent less for the flow including a transition slope than for flow without the slope.

CHAPTER V

DISCUSSION

V-1. Physical Interpretation of Results

Sawyer (1959) classified the motions associated with topographic effects as follows:

(a) Small scale turbulent motion mainly near the ground or ocean bottom.

(b) Gravity oscillations which may extend upward to great heights.

(c) Oscillations about geostrophic equilibrium.

(d) Quasi-geostrophic motions.

The frictional effects denoted by (a) remain largely an unknown factor in large scale oceanic flows. Görtler (1941) utilized the boundary layer principles developed by Prandtl to define an equivalent topography as that relief having the same effect on a frictionless current as does the actual topography on the real frictional current. He then treated the flow in the friction layer as a secondary current flowing normal to the main stream. A detailed analysis by Görtler showed that both the magnitude of the secondary current and the deviation of the equivalent, or effective, topography from reality are usually so slight as to be negligible. Stommel (1957) argued convincingly for the treatment of Ekman convergence and divergence at both the surface and bottom of the sea in the same manner as precipitation and runoff are treated, namely, as external parameters influencing the internal flow regime. In any case, the topographic features under discussion are taken to be large enough that they do not directly

influence the frictional processes. There must be a point, however, at which the topographic disturbance of an originally frictionless flow may become large enough to require the consideration of friction in the subsequent flow. This is likely to be the case for the unstable flow patterns shown in Figures 3b and 6.

By the assumption of homogeneity gravity oscillations of other than the free surface have been precluded, although it is recognized that internal oscillations may be an important, or even a dominant, result of topographic perturbations. If we adopt Sawyer's (1959) definition of gravity oscillations as those with periods much less than a half pendulum day, we readily see that they may be neglected for our steady barotropic model. The shallow water waves associated with the depths of the ocean have phase velocities on the order of 100 m/sec, certainly faster than any current. Conversely, no deep water gravity waves exist with a phase velocity of 10 cm/sec (a reasonable choice for a current velocity). Thus it seems reasonable not to expect any stationary, surface, gravity waves to be associated with topographic effects on deep ocean currents.

The classifications (c) and (d) appear to merge in a barotropic model. It is essential to note that the term "quasi-geostrophic" is a result of the utilization of perturbation techniques, and may be misleading. To illustrate this, Gambo (1957) demonstrated that the quasi-geostrophic approximation is poor for surface slopes of greater than 1/1000, or about 5 minutes in angular measure. This is certainly less slope, by an order of magnitude or more, than is frequently found for both subaerial and submarine topographies. Thus it is believed that an examination of the developments given in the previous three chapters may give some insight into those

topographic effects that lie without the region of applicability of linearized perturbation techniques.

Returning to the fundamental equation for the conservation of potential vorticity, equation (4), the qualitative effects of shoaling water (or deepening water) on an initially uniform zonal flow may be readily seen. For flow to the east, shoaling conditions immediately require a decrease in relative vorticity, or the establishment of anticyclonic flow. This turns the streamline towards lower latitudes where the Coriolis parameter is also decreasing. Depending on the change of depth, a point will be reached where the decrease in the Coriolis parameter will have balanced the change in depth, and subsequent motion towards the equator will cause the flow to become cyclonic thus returning the streamline to its original latitude. The converse is true for eastward flowing currents passing over deepening relief, except that there is no theoretical limit on the magnitude of the depth change, and hence equilibrium may not be reached.

If the flow is <u>from</u> the east however, equation (4) shows that a wave-like motion is not expected, because there is no "restoring force." Shoaling conditions again require the development of anticyclonic flow, which in this case deflects the streamline to higher latitudes, and therefore higher values of the Coriolis parameter. This in turn requires a still further decrease in relative vorticity. The general solution (16) is applicable to flow in either direction. For eastward flows, s is real, giving a streamfunction periodic in s and therefore in x. For westward flows, U is negative, giving an imaginary value for p. This then transforms the trigonometric terms in solution (16) to give an exponential dependence of the streamfunction on x.

This qualitative analysis of the vorticity equation is equally applicable to the Northern and Southern Hemispheres if the y axis is taken poleward, the x axis eastward, and the sign of the relative vorticity is defined as agreeing with the sign of the local vertical component of the earth's rotation.

V-2. Comparison with Previous Investigations

It is interesting to note that the results of this paper may be qualitatively compared with those of Fofonoff (1962) for the steady inertial circulation of a rectangular ocean of uniform depth and barotropic properties. Fofonoff obtained solutions to equation (8) of this paper (without the depth variation) subject to the condition that the boundary be a streamline. His solution for positive U is also periodic in x, but he chose to interpret it as a time-dependent solution superimposed on a steady zonal current.

The occurrence of critical flow patterns was discussed by Bolin (1950). He utilized the perturbation equations to investigate the flow of a very narrow zonal current over a single mountain range. A critical velocity was found for which the flow would return to the source region. That velocity was given as the ratio of the local value of the Coriolis parameter to the total height of the atmosphere, multiplied by the cross sectional area of the mountain range. Although the relevant choice of parameters was at or beyond the limit of the perturbation equations, Bolin demonstrated that such a critical regime might exist for features on the order of the Rocky Mountains. It is apparent that the streamlines depicted for critical flow in Chapter IV do not return to the source region because of the specification of an infinitely wide upstream zonal flow.

V-3. Numerical Evaluation and Comparison

(a) Atmospheric Example

Queney (1949), in his treatment of a stably stratified atmosphere of infinite height, gave the perturbations associated with flow at 10 m/sec over a plateau of height 0.5 km and width 300 km. Although the lack of a finite height of the air column in Queney's analysis hampers comparison, his results may be qualitatively related to those of Chapter IV by accepting Bolin's (1950) statement on the equivalent barotropic atmosphere for the case of stable stratification. Queney found only vertical perturbations of the type (b) above. He stated that lateral deflections are negligible. Using 8 km as the undisturbed height of the atmosphere, and the above dimensions of the plateau, the flow over the plateau is described by equation (17) as

$$-\psi_2(x,y) = 8.5 \times 10^8 y \, (1 - 0.06 \cos 1.3 \times 10^{-8} x).$$

The coefficient of the cosine term in the above equation would seem to indicate a small deflection, but recalling that the y axis originates at the equator, this coefficient would give a deflection of about 6 percent of the original latitude were the plateau broad enough. However, the argument of the cosine term varies through only 22 degrees for a breadth of 500 km, giving a maximum deflection at the leeward edge of the plateau of about 0.4 percent of the original latitude, or about 10 km at 30°N.

From equation (21), downwind of the plateau

$$-\psi_3 = 8.0 \times 10^8 y \left[1 + 0.004 \cos p(x - 500 \text{ km}) + 0.02 \sin p(x - 500 \text{ km})\right].$$

From this equation it is apparent that the amplitude of the downstream deflection is approximately 2 percent of the original latitude, or about 70 km at 30°N. This amplitude must be compared with a representative wavelength of about 5000 km, and therefore can be considered small.

Considering the above example, if the results of Chapter IV are to be applicable to atmospheric flows, the scale of the relief must be larger than that considered by Queney. In fact, it was shown in Chapter IV that a more representative wavelength of a downstream oscillation is on the order of 10,000 km, and it is therefore clear that the only features that will give an oscillation of appreciable amplitude are those having a characteristic zonal dimension that is a significant fraction of the wavelength of the free wave.

(b) Gulf Stream Meanders

If we now attempt to compare the results of Chapters II - IV with observed oceanic currents, we are met by a marked lack of data for the barotropic mode. Rossby (1938) pointed out that, although there will always be a tendency towards the redistribution of mass in the oceans that will reduce horizontal pressure gradients with depth, such adjustments are not immediate and changes in the distribution of wind stress and atmospheric pressure must induce pressure gradients at the floor of the ocean. Such transient barotropic responses have not been included in this paper's steady state model. Recalling that the existence of barotropic flow was postulated earlier, the presence of oxygenated water in the deep ocean basins is accepted as primary evidence that a net flow extends to the depths of the oceans.

Swallow (1957) found 0.8 cm/sec as the average velocity at about 3000 m depth in the Northeast Atlantic. Volkmann (1962) measured deep currents on the order of 10 cm/sec at 3000 m just inshore of the Gulf Stream south of Cape Cod. Defant (1961) summarized earlier calculations of relative currents in the deep layers of the Atlantic. These calculations show velocities varying from 0 to 20 cm/sec over a range of depths from

2000 m to near the bottom. The wavelengths of Stationary Rossby Waves associated with these velocities are on the order of a few hundred kilometers, certainly well within the existing scales of bathymetric relief in the deep oceans. It would therefore seem clear that the effects of topography on deep currents in the ocean should be appreciable over a much broader range of scales than those found to be important in atmospheric flows.

The solution (16) may be evaluated, if only numerically, for any relief of sufficient interest. In view of the rather restrictive assumption of an initially uniform zonal flow, and the lack of data on in situ currents, such an evaluation should await further refinement of the theory. One problem that could provide sufficient interest for future evaluation is that of the Gulf Stream; specifically, the problem of the meanders that appear after the Stream departs the Blake Plateau.

Stommel (1960) discussed the meanders of the Gulf Stream at length. He utilized a perturbation technique to argue tentatively that the instability of such meanders may be associated with a critical internal Froude number. As another possibility he showed that the potential vorticity of the Stream is constant over a cross section, and thereby proceeded to develop a linearized theory of stable meanders. Stommel also mentioned the possibility that sea mounts may influence the development of the meanders, but did not develop a mechanism for such an influence.

Such meanders are characteristically about 200 km in wavelength. Although any connection between the developments in Chapters III and IV and the real highly baroclinic Gulf Stream are at this stage tenuous at best, it is interesting to calculate the barotropic flow that would be associated with meanders of the dimensions observed. For L = 200 km, and again taking $\beta = 2 \cdot 10^{-13} \text{sec}^{-1} \text{cm}^{-1}$, the numerical relationship

$r_2 = 0.5\ U^{\frac{1}{2}}$(cm/sec) is obtained. Observing a rather rapid increase in depth from 1000 m to 3000 m in the region where the Stream leaves the Blake Plateau, r_2 is taken as roughly 3, in keeping with the introductory remarks concerning the utilization of realistic bathymetry. The instability associated with this choice of r_2 will be neglected initially, but will be discussed in a subsequent section. Such a change of depth would produce meanders of the observed wavelength in a barotropic current flowing at 18 cm/sec. Volkmann (1962), although primarily interested in the apparent counter-current inshore of the Gulf Stream, showed (Volkmann, Fig. 3) the 18 cm/sec isopleth at a depth of 2200 m with flow in the same direction as the Stream. The location is well removed from Cape Hatteras, being at approximately 36°N - 68°W, but may indicate strong barotropic currents in the Gulf Stream region.

A more moderate change of depth would require a smaller velocity. That such physically reasonable velocities are required despite the rather serious violation of the initial assumption of a uniform zonal flow indicates that such an assumption may not be too essential.

As has been noted above, there is at present no convincing physical argument that allows the supposition of an equivalent barotropic Gulf Stream. Only if there is a degree of coupling between the surface Stream and the flow at depth can the above results be interpreted as indicating a possible cause and effect relationship. It seems reasonable to infer, however, that if a barotropic flow exists independent of the Gulf Stream, and it is subjected to the above described change in depth, the surface current may be influenced by the flow of the deeper layer.

(c) South Atlantic

We now turn our attention to the South Atlantic and attempt to compare the results of Chapter IV with some of the observed features of the dynamic topography considered by Shtokman (1947) (see Figure 2). Retaining the Lagrangian viewpoint, Figure 2 shows an eastward flow departing the broad continental shelf in the vicinity of 40°S. The subsequent southerly excursion of the transport isopleths is approximately centered over the rather broad Argentine Basin (depth > 5000 m). This is in qualitative agreement with the results of Chapter IV, which give poleward deflection over deepening relief.

If, as for the Gulf Stream, a relative depth of 3 is assumed (again neglecting for the moment the instability implied by equation (18)), and observing that the wavelength along 40°S is about 8 degrees of latitude (about four times that of the Gulf Stream meanders), the velocity required is found to be 16 times that required for the barotropic component of the Gulf Stream, or about 3 m/sec. This certainly bears no relation to physical reality.

Similarly, downstream of the Argentine Basin the wavelengths of the dynamic contours remain on the order of 10 degrees of latitude, which for deepening relief require unrealistic velocities. Shoaling conditions however (say $r \approx \frac{1}{2}$), could give realistic velocities for such wavelengths. The complexity of the bathymetry and of the upstream flow pattern precludes a realistic evaluation utilizing the model of Chapter III - IV. It should be noted that the apparent discrepancy cited by Shtokman (1947) between models in which the direction of the deflection depends only on the hemisphere, and the dynamic topography shown in Figure 2, is subject to re-interpretation. Considering the region over the Mid-Atlantic Ridge at 40°S - 10°W, Shtokman

felt that the general southerly set of the current over gradually shoaling water contradicts Ekman and Sverdrup. Defant (1961) however noted that the sharp anti-cyclonic curvature of the isopleths of dynamic height approximately over the top of the Ridge substantiates the Coriolis parameter models. The solutions obtained in the previous chapters of this paper of course require equatorward deflection over shoaling water for upstream zonal flow, but do not explicitly treat the probable interactions between the effects of successive bathymetric features.

(d) Antarctic Circumpolar Current

The Antarctic Circumpolar Current (ACC) affords the classic example of topographic effects on zonal currents (see Figure 1). In general the relative (baroclinic) currents are observed to be deflected towards the equator over shoaling water and towards the pole over deepening water, an observation which qualitatively agrees with our results. The sharp northward perturbation of the isopleths over the New Zealand Plateau will be considered first. The apparent lack of any downstream effects indicates that the Plateau should be about one wave length long, if the results of Chapter IV are to be applicable. The relative depth associated with the Plateau is about 0.4 and a characteristic width at 50°S is 780 km. These figures require a barotropic velocity of about 5 cm/sec, a reasonable value.

The flow of the ACC over the Scotia Ridge and South Sandwich Trench system affords an even more striking example of topographic effects, although the constrictive effects of Drake Passage tend to complicate the upstream flow pattern. The complexity of the bathymetry in this region, including the marked cross-stream variations in depth associated with the South Sandwich and Orkney Islands and South Georgia precludes all but the roughest

estimates of relative depth and feature breadth. Such an estimate however does yield a physically reasonable velocity on the order of 10 cm/sec.

Between 60°E and 90°E Figure 8 shows that the ACC is deflected towards the equator as it passes over the Kerguelen - Gaussberg Ridge. Again utilizing the final investigation of Chapter IV to validate the approximation of such a smooth feature by a plateau, the distance between the 3600 m isobaths along 55°S is chosen as the representative width; about 550 km. Representative depths of 4.5 km upstream and 3.6 km over the feature with a relative depth of 0.8 are chosen. These values of the parameters require a barotropic velocity of 9.8 cm/sec for undisturbed downstream flow.

V-4. Comparison and Discussion of Amplitudes

It would be advantageous to be able to use the amplitudes of the observed disturbances as independent checks of the agreement between theory and observation. Agreement by this means can not be obtained for the examples cited above. As has been noted previously, the relative depth of 3 assumed for the Gulf Stream and for the Argentine Basin examples give in each case unstable flow conditions for the frictionless model. The amplitude of the disturbance over the New Zealand Plateau should be approximately 40 percent of the original latitude, or about 20 degrees, rather than the observed 5 - 10 degrees. The calculated amplitude over the Kerguelen - Gaussberg Ridge is about 17 degrees as opposed to the observed amplitude of 7 - 8 degrees.

There are several possible explanations for the discrepancies between the amplitudes of the observed and calculated disturbances. It was shown at the end of Chapter IV that the mode of transition in depth from one region to another has little effect on the amplitude of the disturbances. Therefore

the step-shaped features considered are not the primary source of the disagreement. The departure of the real current from the assumption of meridional uniformity is certainly appreciable in the case of the Gulf Stream, but would seem to be minor in the case of the Antarctic Circumpolar Current.

Of the two remaining major factors, friction and baroclinicity, it seems probable that the latter is of fundamental importance in determining the amplitude of the topographic disturbance. Saint-Guily (1962) noted that this difference in amplitude is likely due " . . . to the existence of a vertical density gradient. Such a gradient apparently has the effect of intensifying the currents near the surface, therefore of reducing the equivalent depth H, ... , and the amplitude of the deflection." Our introductory postulation of the existence of a barotropic mode of oceanic flow included no a priori expectation that such a flow over physically realistic topographic features would directly correspond to the observed relative dynamic topography. It might be expected that an equivalent mode of circulation with uniform velocities extending to great depths would also require the introduction of the concept of an equivalent topographic feature. Unfortunately there is as yet no independent procedure available by which the relief of such a feature may be inferred.

In the brief examples given on previous pages of the chapter the observed wavelengths and the actual bathymetry have been utilized to calculate barotropic velocities. This approach may be modified and both observed wavelengths and observed amplitudes utilized to calculate equivalent velocities and topographies. For the flow over the New Zealand Plateau shown in Figure 1, the amplitude is about 10 degrees of latitude and the wavelength is about 780 km. Substitution of these values into equation (18)

yields an equivalent relative depth r_2, of about 0.9 (as opposed to the physical change in depth of about 0.4). The resulting barotropic velocity is about 25 cm/sec.

A similar evaluation of the flow over the Kerguelen - Gaussberg Ridge yields an equivalent relative depth of 0.94 and a barotropic velocity of 14 cm/sec.

A characteristic amplitude of the Gulf Stream meanders is less easily defined, but as a rough approximation it may be taken to be of the same order as the wavelength, about 200 km (Stommel, 1960). This value gives a barotropic velocity of 2 cm/sec, and the very small relative depth of 1.03.

The above consideration of an equivalent topography indicates the reason for the qualitative success of the perturbation theories. Such theories would be seemingly justified on the basis of the small effective changes of depth that were above found to be required to produce disturbances of the barotropic mode that correspond to the observed disturbances of the baroclinic component of the ocean currents. Such a distinction between actual and effective topography has not generally been made.

CHAPTER VI

SUMMARY

1. An exact solution has been obtained for the steady disturbance of an initially uniform, frictionless zonal barotropic current by a meridionally uniform topographic feature.

2. The resulting streamlines are zonally periodic for eastward flow and exponential for westward flow.

3. In both the Northern and Southern Hemispheres eastward flow is deflected towards the equator over shoaling water and towards the poles over deepening water.

4. Topographic effects are not limited to the region of variable depth, but may exist downstream in the form of a standing Rossby Wave.

5. For a finite decrease in depth between two regions, the manner of transition in bathymetry is unimportant and may be treated as discontinuous.

6. The wavelengths of stationary Rossby waves associated with characteristic velocities of the deep currents in the ocean are such that the existing scales of topographic features should be highly effective in disturbing deep barotropic currents.

7. Very small changes in depth in the barotropic model are sufficient to produce disturbances of the dimensions observed in the real ocean.

8. Future study should consider the effects of the baroclinicity of the ocean, which should be properly related to both the actual topography and the vertical structure of the oceanic currents.

BIBLIOGRAPHY

BERGGREN, R., BOLIN, B., and ROSSBY, C.-G. (1949) An aerological study of zonal motion, its perturbations and breakdown. Tellus 2 (1), 14-37.

BOLIN, B. (1950) On the influence of the earths orography on the general character of the westerlies. Tellus 2 (3), 184-195.

CHARNEY, J. G. (1955) The Gulf Stream as an inertial boundary layer. Proc. Nat. Acad. Sci. 41, 731-740.

CHARNEY, J. G. and ELIASSEN (1949) A numerical method for predicting the perturbations of the middle latitude westerlies. Tellus 1 (2), 38-54.

DEFANT, A. (1941) Die absolute Topographie des physikalischen Meeresniveaus und der Druckflachen, sowie die Wasserbew egungen im Atlantischen Ozean. in "Deutsche Atlantische Expedition Meteor 1925-1927," Wissenschaftliche Ergebnisse, Bd 6, (2) 191-260.

DEFANT, A. (1961) Physical Oceanography, Vol. 1. Pergamon, New York. 682-693.

EKMAN, V. W. (1923) Über horizontalzirkulation bei winderzeugten meeresströmungen. Ark. Math., Astr. och Fys. 17 (26), 60-63.

EKMAN, V. W. (1932) Studien zur dynamik der meeresströmungen, Gerlands Beitrage zur Geophysik , 36, 430-438.

FOFONOFF, N. P. (1962) Dynamics of ocean currents. in The Sea, vol. 1, M. N. Hill, ed. Interscience, New York, 349-354.

FUGLISTER, F. C. and WORTHINGTON, L. V. (1951) Some results of a multiple ship survey of the Gulf Stream. Tellus 3 (1), 1-14.

GAMBO, K. (1957) The scale of atmospheric motions and effect of topography on numerical weather prediction in the lower atmosphere. Pap. Met. Geophys., Tokyo, 8, 1-24.

GÖRTLER, H. (1941) Einfluss der Bodentopographie auf Strömungen über der rotierenden Erde. Z. Angew. Math. Mech. 21, 279-303.

NEUMANN, G. (1960) On the effect of bottom topography on ocean currents. Dt. Hydrogr. Z. 13, 132-141.

OI, M. (1956) Topographic perturbation of a zonal current. Meteorol. Notes Met. Res. Inst., Kyoto Univ., Series 2, No. 10, 16-31.

QUENEY, P. (1947) Theory of perturbations in stratified currents with applications to air flow over mountain barriers. Dept. of Meteor., Univ. Chicago Misc. Rep. No. 23, 1-81.

QUENEY, P. (1949) The problem of air flow over mountains; a summary of theoretical studies. Bull. Am. Met. Soc. 29 (1), 16-26.

ROSSBY, C.-G. (1938) On the mutual adjustment of pressure and velocity distributions in certain simple current systems, II. J. Mar. Res. 1 (3), 239-262.

SAINT-GUILY, B. (1959) Sur la solution du probleme d'Ekman. Dt. Hydrogr. Z. 13, 262-270.

SAINT-GUILY, B. (1962) On the general form of the Ekman Problem. Proc. Symp. Math.-Hydrodynamical Methods of Physical Oceanography. Sept. 1961. Institut fur Meereskunde der Universitat Hamburg, 61-75.

SAWYER, J. S. (1959) The introduction of the effects of topography into methods of numerical forecasting. Quart. J. R. Met. Soc. 85 (363), 31-43.

SHTOKMAN, V. B. (1947) Effect of bottom topography on the direction of currents in the sea. Priroda 11, 10-23. (English transl. by E. R. Hope in Defence Res. Bd. Canada, T 57 R, 20 pp., 1952)

SHTOKMAN, V. B. (1948a) Relationships between the wind field, the transport field and the average field of mass in a non-homogeneous ocean. Dokl. Akad. Nauk SSSR, 59 (4), 675-678. (English transl. by E. R. Hope in Defence Res. Bd. Canada, T 56 R, 7 pp., 1952)

SHTOKMAN, V. B. (1948b) Effect of bottom topography on the direction of the transport of water set up by the wind or the mass field in a non-homogeneous ocean. Dokl. Akad. Nauk SSSR, 59 (5), 889-892. (English transl. by E. R. Hope in Defence Res. Bd. Canada, T 57 R, 20 pp., 1952)

STOMMEL, H. (1957) A survey of ocean current theory. Deep-Sea Res. 4, 149-184.

STOMMEL, H. (1960) The Gulf Stream. Univ. of Calif. Press, Berkeley.

SVERDRUP, H. U. (1941) The influences of bottom topography on ocean currents. Applied Mechanics, Th. v. Kármán Anniv. Vol., 66-75.

SVERDRUP, H. U., JOHNSON, M. W. and FLEMING, R. H. (1942) The Oceans. Prentice-Hall, New York, 468.

SWALLOW, J. C. (1957) Some further deep current measurements using neutrally buoyant floats. Deep-Sea Res. 4, 93-104.

VOLKMANN, G. (1962) Deep current measurements in the Western North Atlantic. Deep-Sea Res. 9, 493-500.

FIGURES

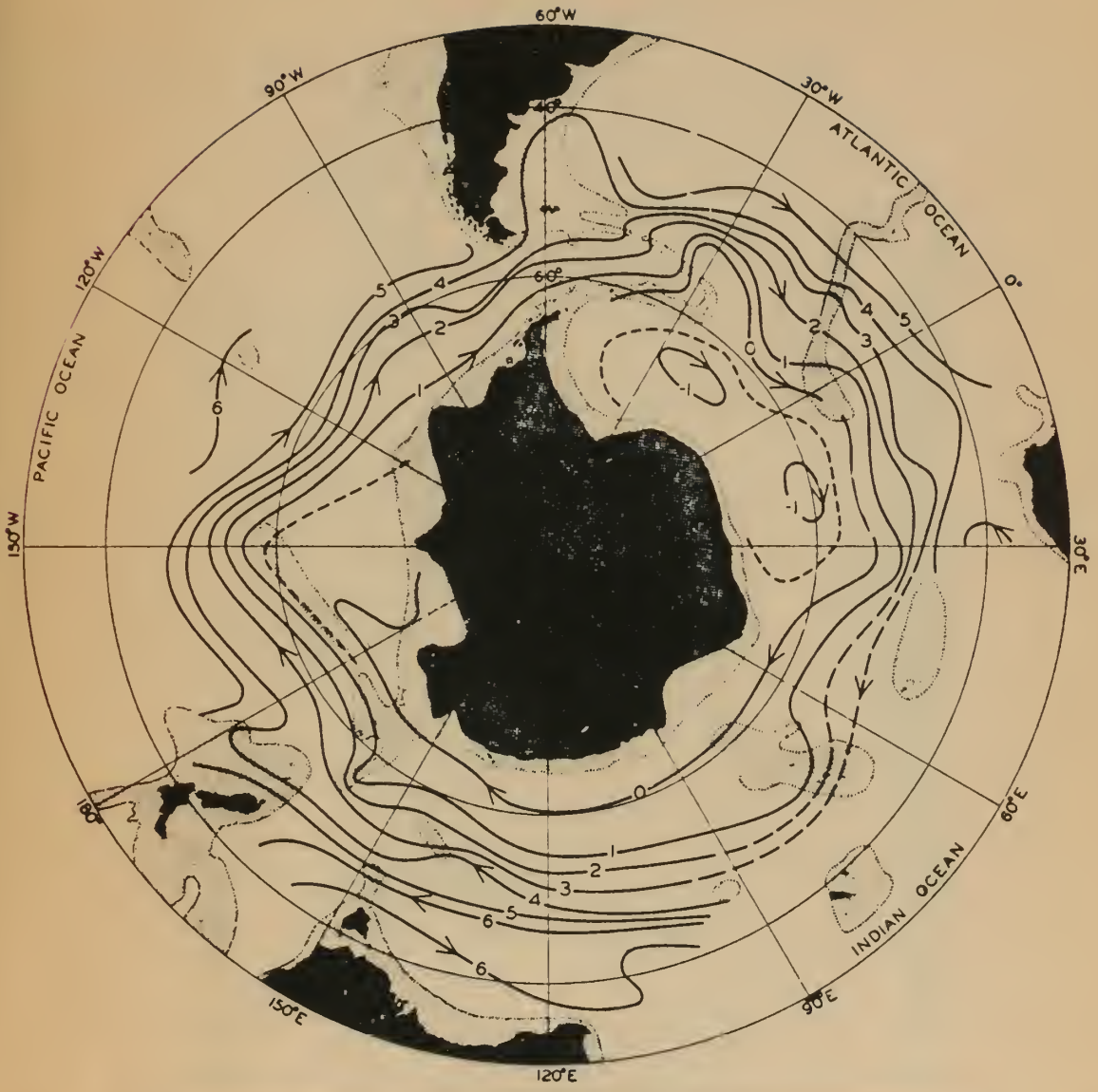


Figure 1

Transport lines around the Antarctic Continent (Sverdrup, et al., 1942)

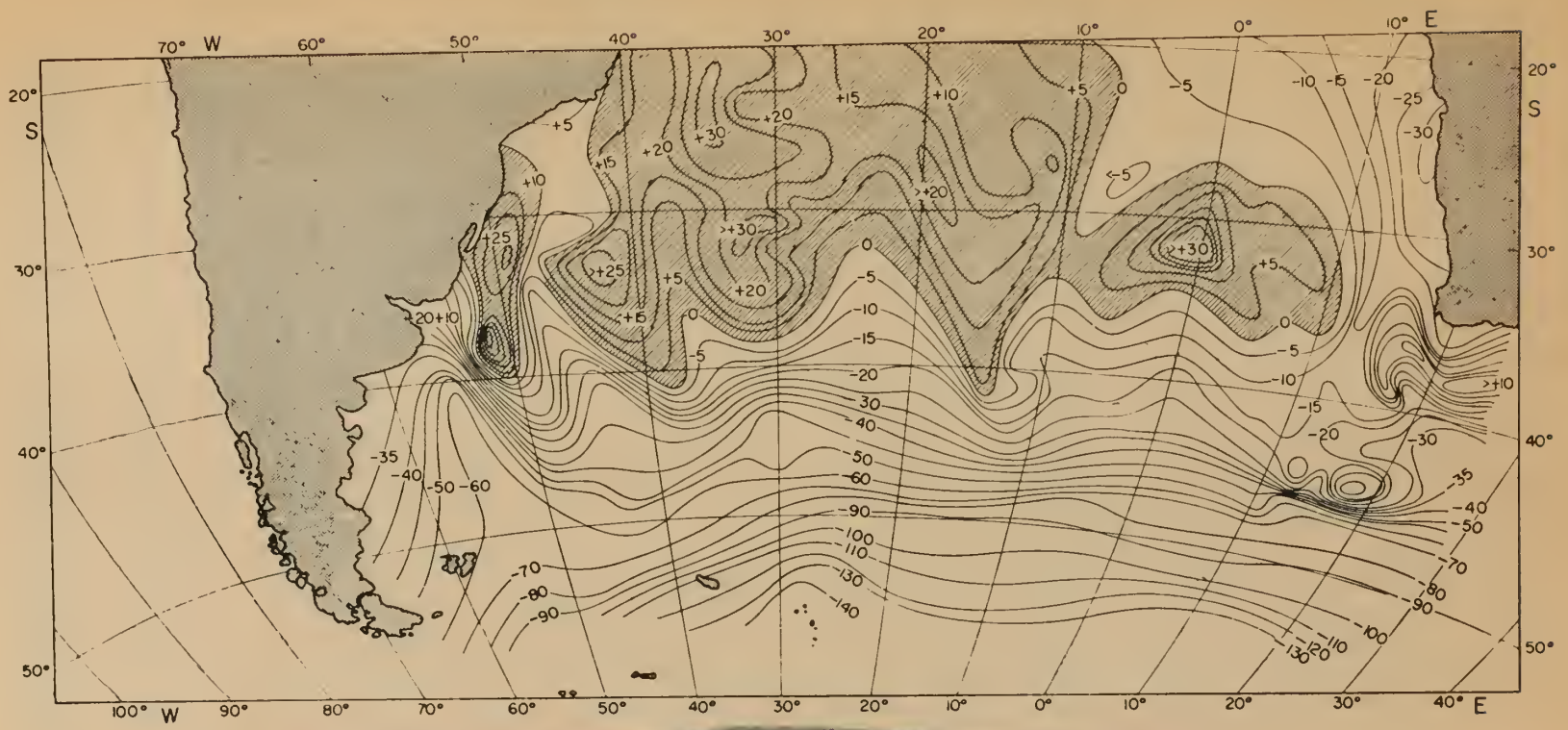
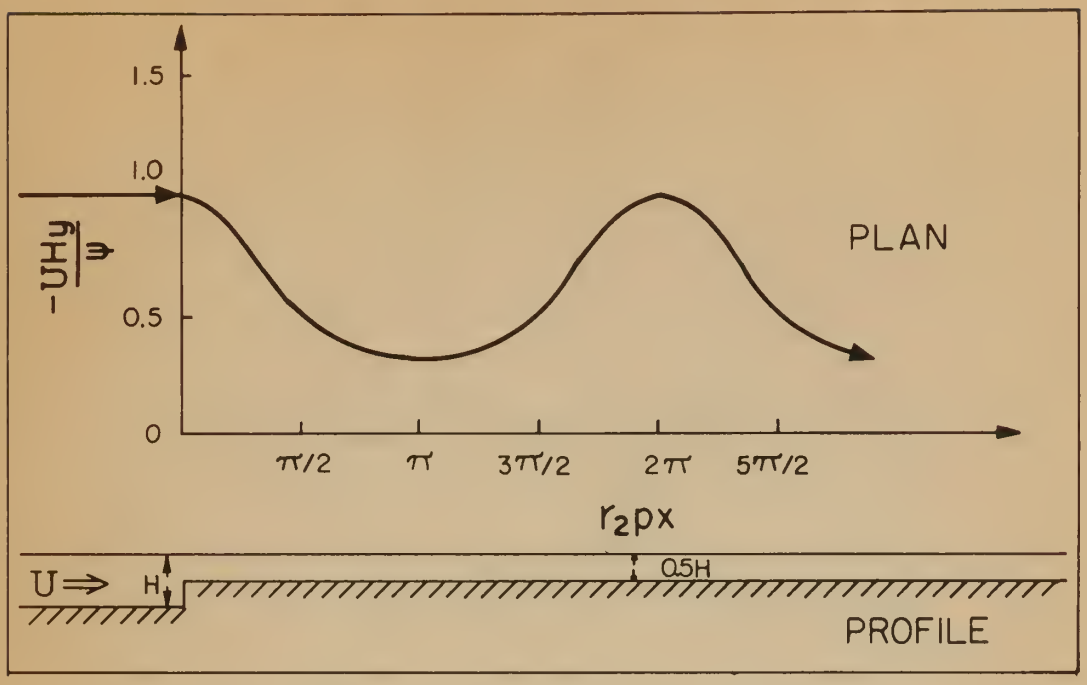


Figure 2

Dynamic topography of the South Atlantic (Defant, 1961)

Figure 3a. Zonal flow over a shoaling step



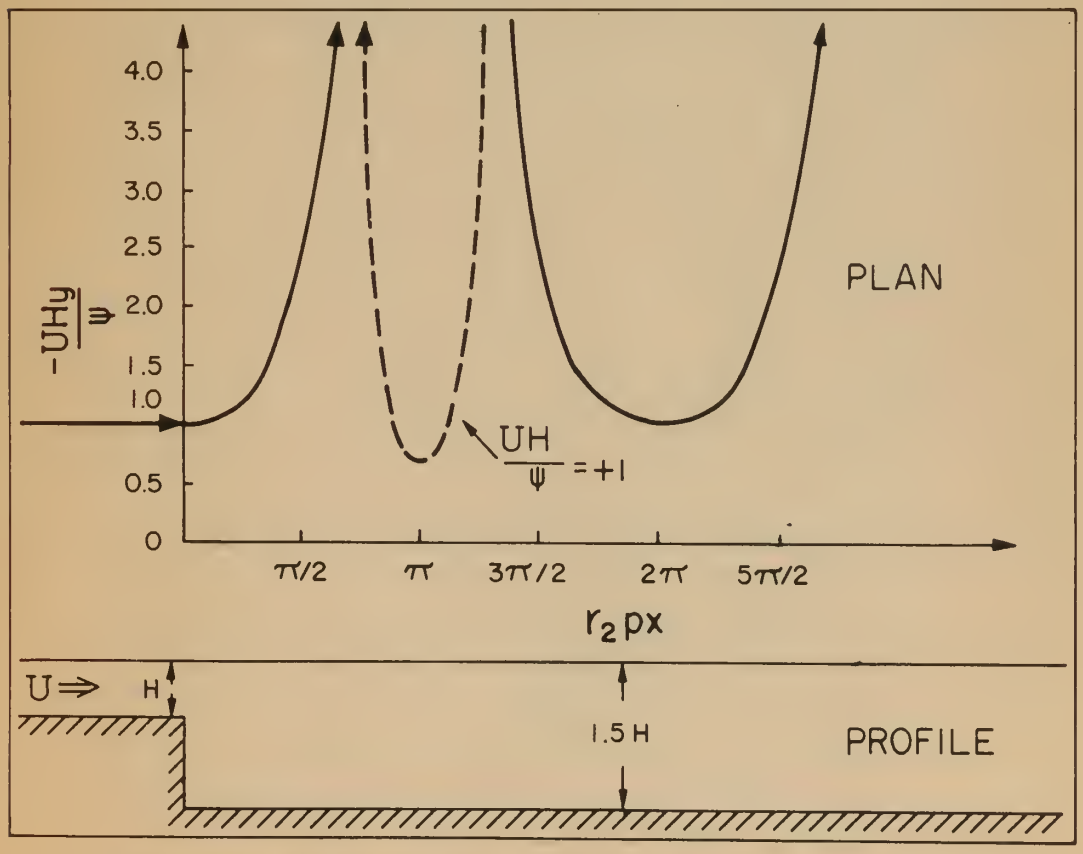


Figure 3b. Zonal flow over a deepening step.

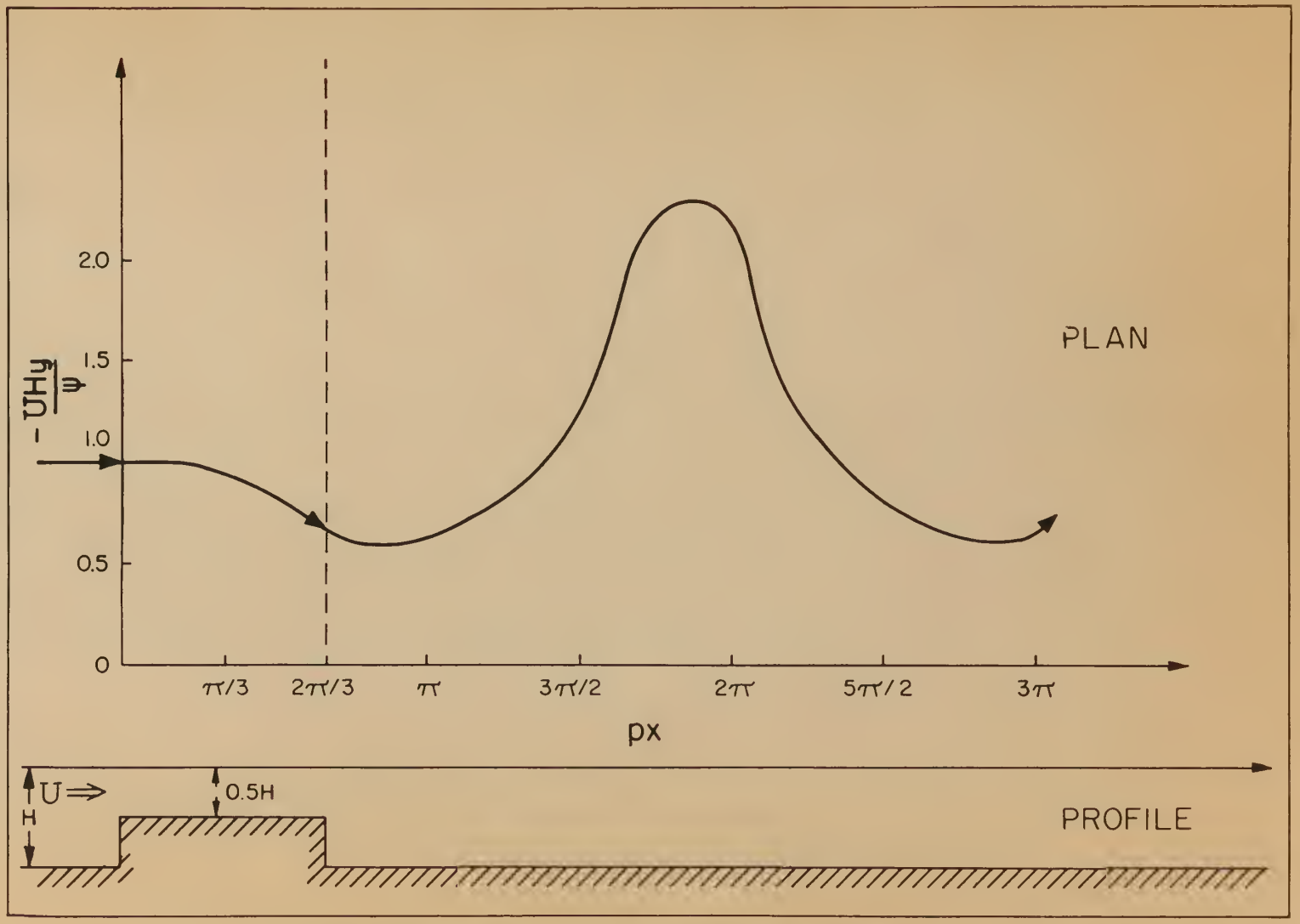


Figure 4. Zonal flow over a narrow plateau

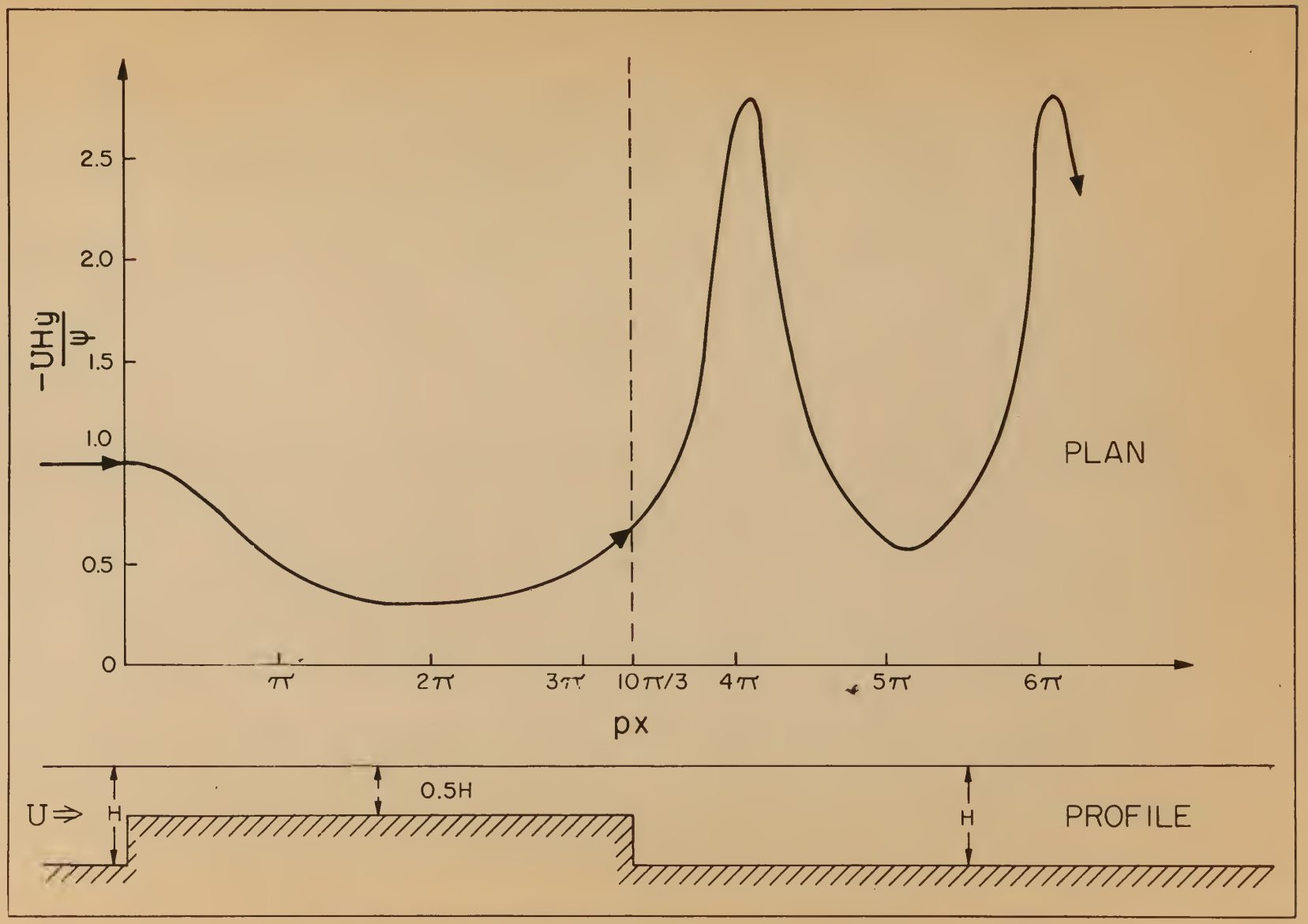


Figure 5. Zonal flow over a wide plateau

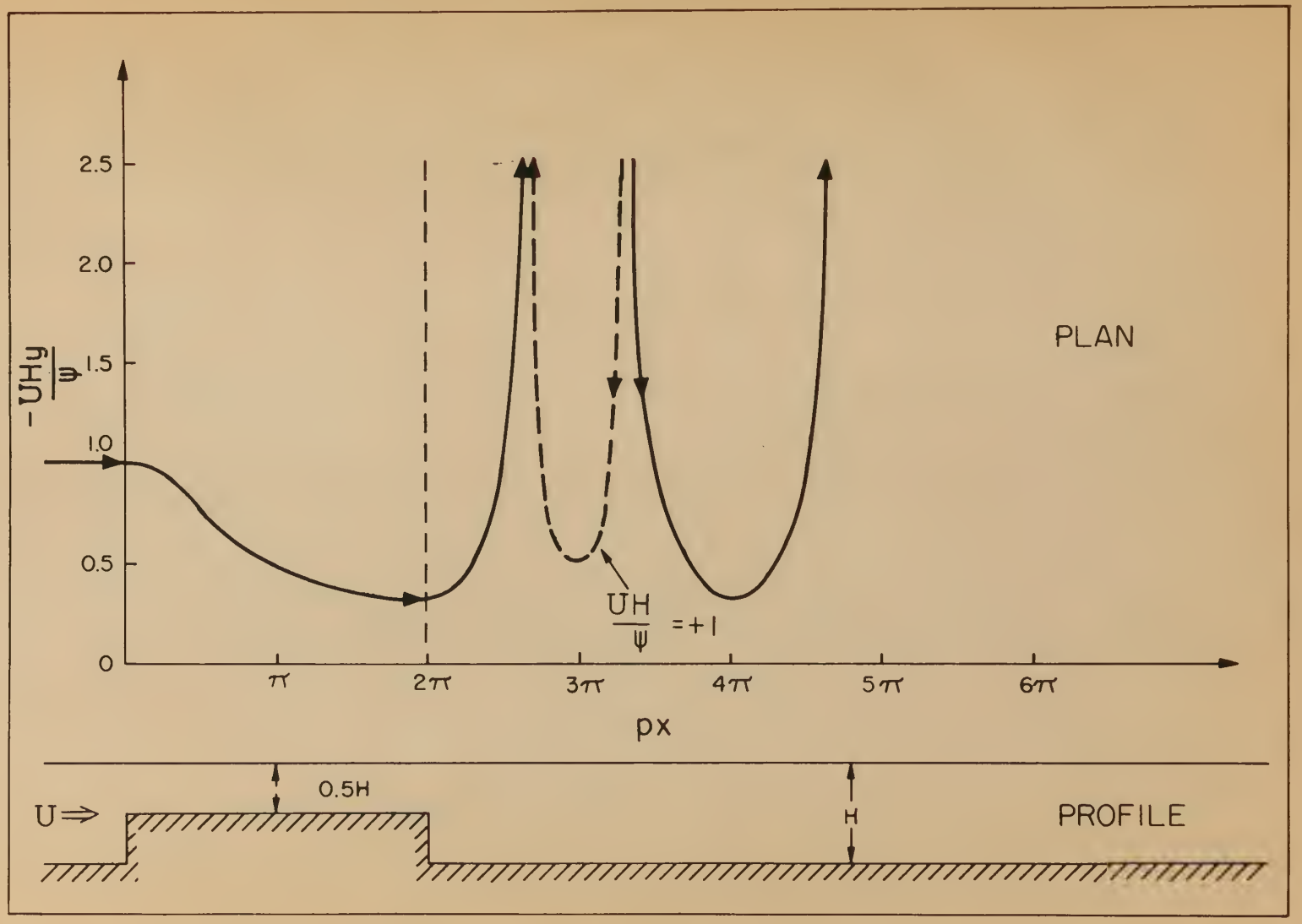


Figure 6. Unstable zonal flow over a plateau

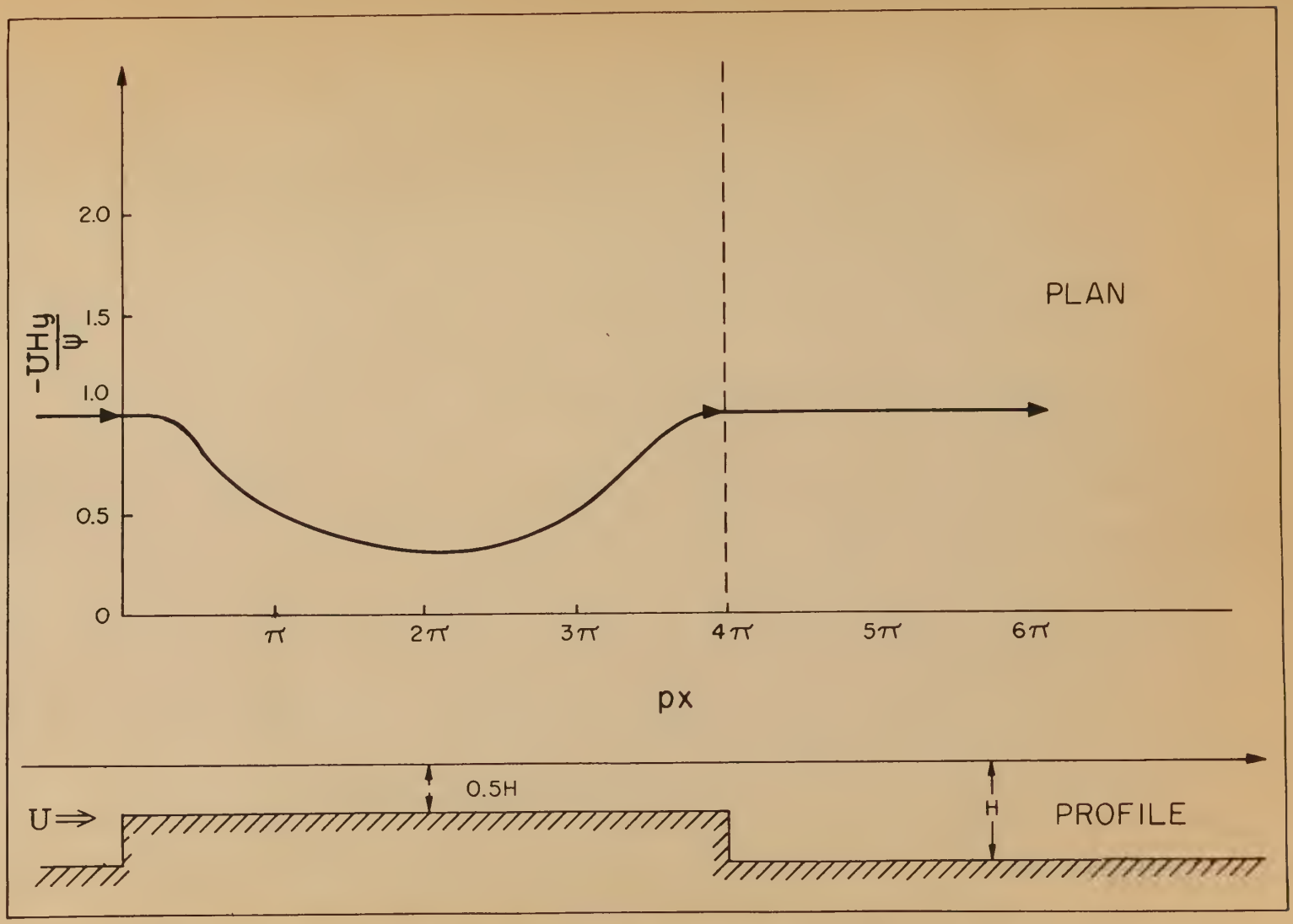


Figure 7. Zonal flow over a plateau (width: 1 wavelength)

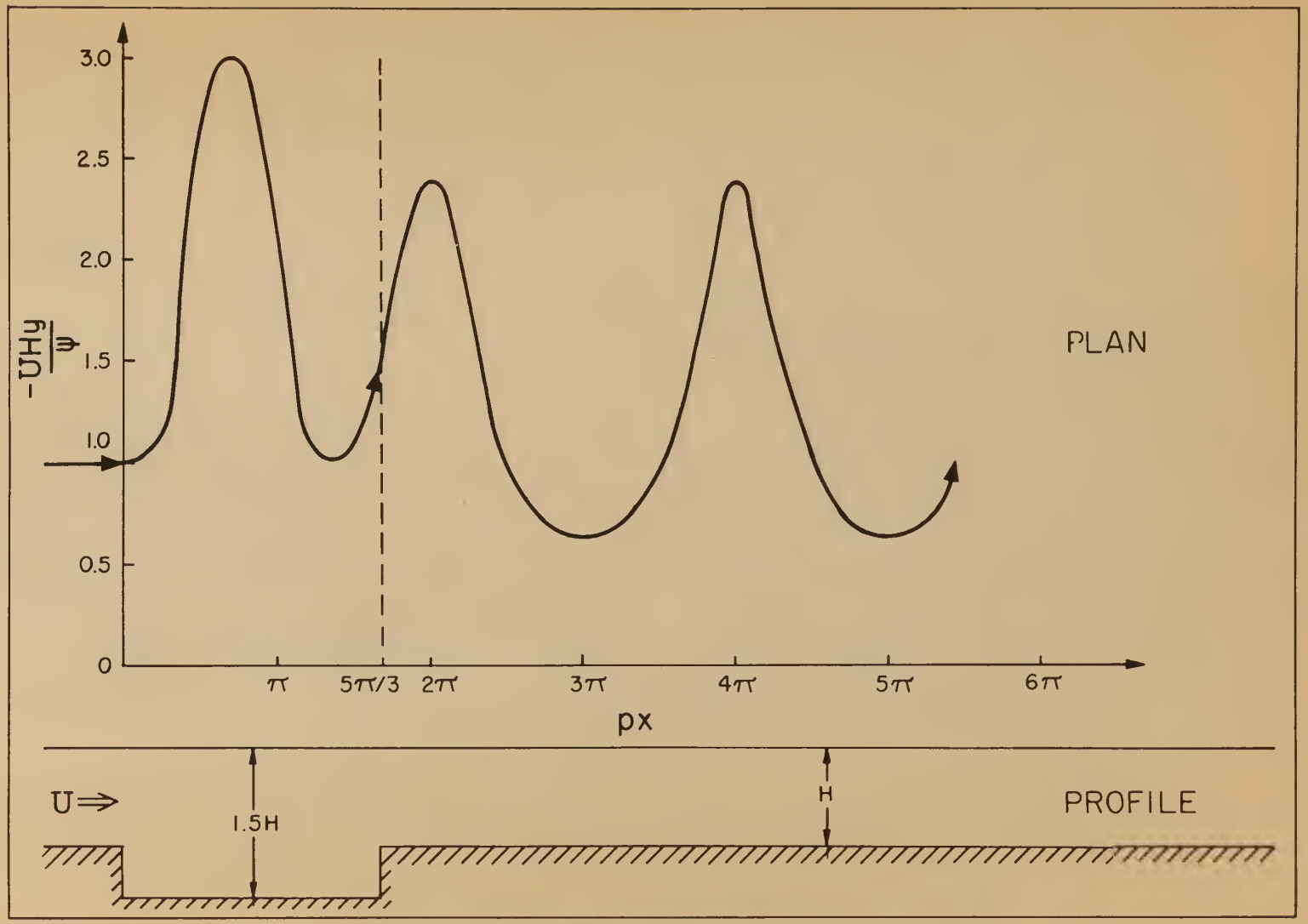


Figure 8. Zonal flow over an idealized valley

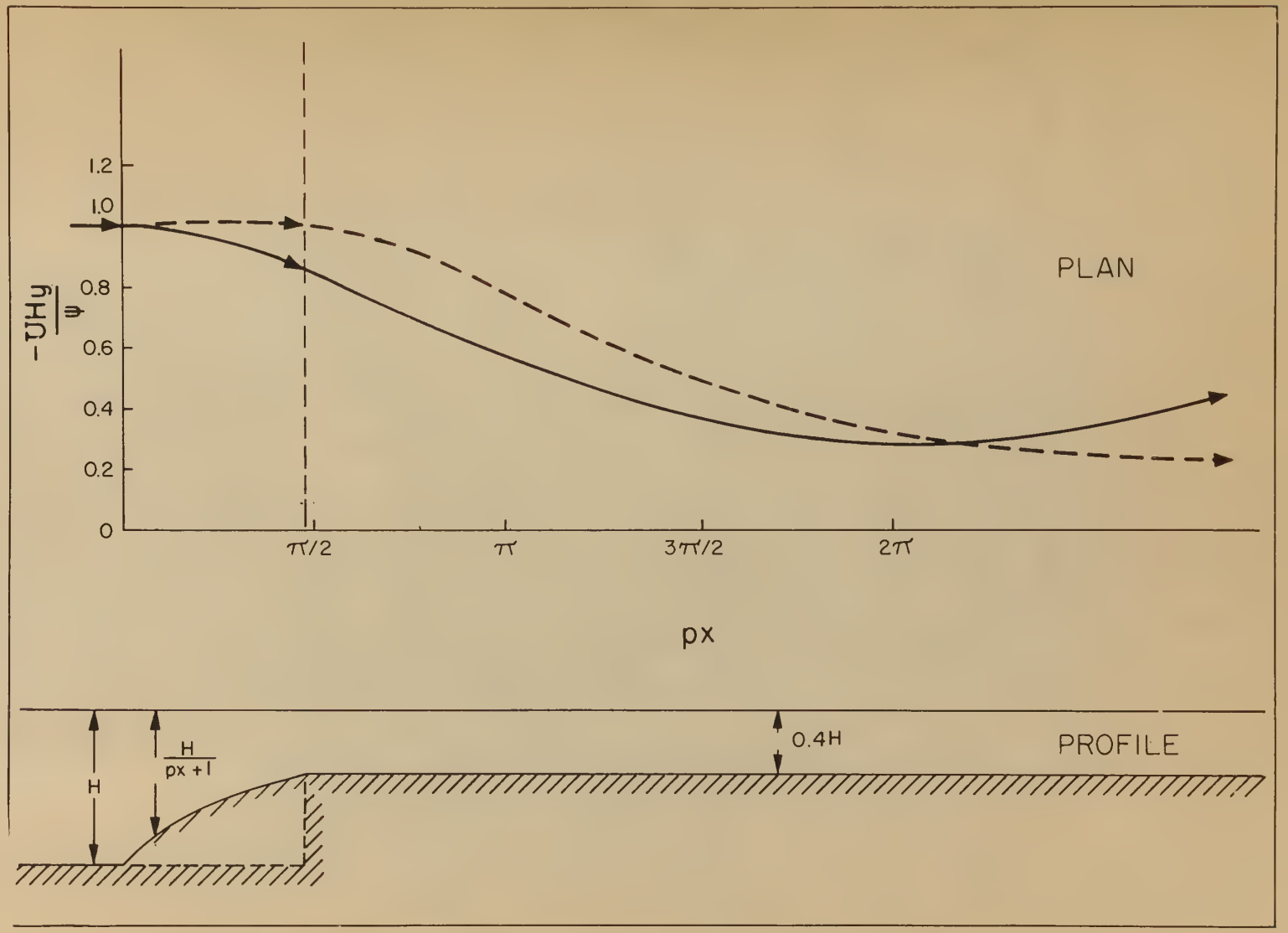


Figure 9. Zonal flow over a transition slope

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